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Multipulse excitability in injected lasers

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ABSTRACT

We show that a single-mode semiconductor laser subject to optical injection, and described by rate equations, can produce excitable multipulses, where the laser emits a certain number of pulses after being triggered from its steady state by a single perturbation.

This phenomenon occurs in experimentally accessible regions in parameter space that are bounded by curves of n -homoclinic bifurcations, connecting a saddle to itself only at the n -th return to a neighborhood of the saddle. These regions are organised in what we call ‘homoclinic teeth’ that grow in size and shape with the linewidth enhancement factor.

Keywords: Semiconductor laser, optical injection, excitability, homoclinic bifurcations

1. INTRODUCTION

When a system is at rest, but reacts to a small but sufficiently large perturbation by producing a large nonlinear response, then it is said to be *excitable*. The minimal size of the perturbation that triggers a response is called the *excitability threshold*. Generally, the response of the system (often in the form of a pulse) is much larger than the perturbation. Furthermore, in many instances the size of the response is largely independent of the size of the perturbation, as long as the perturbation is above the excitability threshold. After having been excited, the system is not excitable for a period of time called the *refractory period*, when the dynamics settles back to the rest state.

Excitability was first introduced in biology to explain spiking of nerve cells¹ and was later also discovered in reaction-diffusion systems.² Recently, excitability of laser systems has been receiving considerable interest. It has been found in:

- nonlinear cavities with temperature dependent absorption,³
- lasers with optical injection,^{4, 5}
- lasers with optical feedback,^{6, 7}
- multisection DFB lasers,^{8, 9}
- lasers with integrated dispersive reflectors,¹⁰ and
- lasers with saturable absorber.^{11, 12}

See also Ref. [13] and the overview Ref. [4].

While being interesting in itself, excitability in lasers also has possible applications. For example, the laser could be used as an optical switch that reacts only to sufficiently high optical input signals. This could be used for pulse reshaping in optical communication, where the idea is that a dispersed input pulse triggers a ‘clean’ output pulse.

In this paper we consider a semiconductor laser subject to optical injection and show that there exist regions, in the plane of the strength K of the injected light and the detuning ω , where the response to a single stimulus may have a form of a fixed number of pulses. How many pulses are produced depends critically on the exact operating conditions; see already Fig. 1. This effect, which we call *multipulse excitability*, was first reported in Ref. [5]. Multipulse excitability appears in regions bounded by curves of n -homoclinic bifurcations, where the homoclinic orbit (to a saddle-focus) closes up only after n global loops; see already Fig. 2. These regions appear inside structures that we call *homoclinic teeth*; see already Fig. 3.

The homoclinic teeth are located inside the laser's locking range and are attached to a curve of saddle-node bifurcations at special bifurcation points (of codimension two) called *noncentral saddle-node homoclinic bifurcations*. This type of bifurcation was shown in Ref. [13] to act as organising centers for excitability in laser systems. Further, we find other special bifurcation points (of codimension two) called *Belyakov bifurcation points*,^{14,15} which are important for organizing the curves of n -homoclinic bifurcation that bound regions of multipulse excitability.

We use the numerical continuation package AUTO¹⁶ to compute the relevant bifurcation curves, in particular the curves of n -homoclinic bifurcations. The phase portraits and time series in Fig. 1 were found with the package DsTool.¹⁷

The paper is organized as follows. In Sec. 2 we introduce the rate equations describing a semiconductor laser with optical injection. Sections 3 deals with the laser's possible response to a single perturbation. In Sec. 4 we show the structure of bifurcation curves responsible for the occurrence of multipulse excitability. Finally, we draw some conclusions in Sec. 5.

2. SEMICONDUCTOR LASER SUBJECT TO OPTICAL INJECTION

A (single-mode) class-B laser receiving optically injected light is a laser system of fundamental and technological importance; see Ref. [18] and further references therein as an entry point to the extensive literature. This system is described well by the rate equations

$$\begin{aligned}\dot{E} &= K + \left(\frac{1}{2}(1 + i\alpha)n - i\omega\right) E \\ \dot{n} &= -2\Gamma n - (1 + 2Bn)(|E|^2 - 1)\end{aligned}\tag{1}$$

for the complex electric field amplitude $E = E_x + iE_y$ and the population inversion n . As was already mentioned, there are two experimentally controllable parameters, namely the injected field rate K and its detuning ω (measured in Eqs. (1) in units of the characteristic relaxation oscillation frequency ω_r) from the free-running laser frequency. For semiconductor laser the linewidth enhancement factor α is larger than zero (typically in the range 1 to 10), and we fix the (rescaled) photon life time B and the (rescaled) damping rate Γ to the realistic values $B = 0.015$ and $\Gamma = 0.035$.

Very good agreement has been shown between the dynamics in Eqs. (1) and experimental measurements, both at local and global scale.^{19–22}

3. REACTION TO A SINGLE PERTURBATION

Figure 1 shows the reaction of system (1) to a single, small perturbation above the excitability threshold. Depending on the exact values of K and ω the result is a single (a), double (b) or triple pulse (c). Here we set $\alpha = 1.0$, but multipulse excitability can be found over a wide range of α -values; see also Sec. 4.

The mechanism of exciting several pulses is the following. There is an attracting steady state (corresponding to locked laser output) and a nearby saddle steady state. The upper branch of the unstable manifold W^u of the saddle makes n loops before ending up at the stable steady state; see Figure 1(d) to (f). A single, very small perturbation of the stable steady state does not have an effect. However, a sufficiently strong perturbation kicks the system to the other side of the stable manifold W^s of the saddle. The result is that the dynamics essentially follows the shape of the unstable manifold W^u back to the stable steady state, resulting in an n -pulse response.

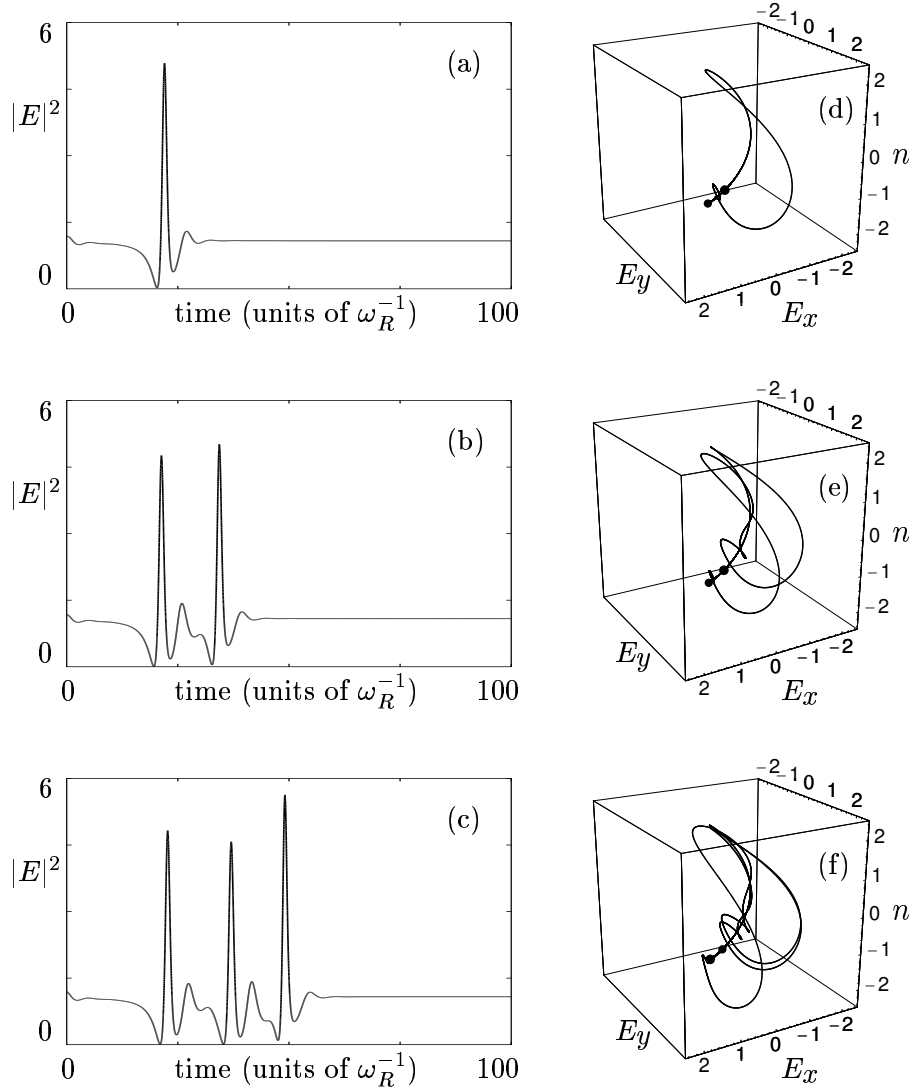


Figure 1. Depending on the exact values of the parameters K and ω , a single perturbation can trigger a multipulse. Panel (a) to (c) show a single, a double and a triple pulse, and panels (d) to (f) the respective trajectory in phase space. From (a) to (c) $(K, \omega) = (0.71, -0.95)$, $(0.745, -1.0)$ and $(0.735, -0.993)$, and $\alpha = 1.0$.

As can be seen, subsequent pulses in Figure 1(b) and (d) are very similar, because the respective trajectories almost retrace themselves a fixed number of times before ending up at the stable steady state. This happens because the system is close to an n -homoclinic bifurcation. This topological configuration is sketched in Fig. 2 for the example of a 1-homoclinic orbit (a) and a 2-homoclinic orbit (c). For parameters near these homoclinic orbits we find 1-pulse and 2-pulse excitability, respectively, as is sketched in panels (b) and (d).

The excitability threshold is given by the stable manifold W^s of the saddle point (which in system (1) is a saddle-focus as sketched). In other words, the system is more excitable if the saddle and the attracting steady state are closer together, that is, if the system is close to a saddle-node bifurcation. The codimension-two

bifurcation point where a curve of n -homoclinic bifurcations meets a saddle-node bifurcation curve is called a noncentral saddle-node homoclinic bifurcation. As we will see now, it acts as an organising center for (multipulse) excitability; see also Ref. [13].

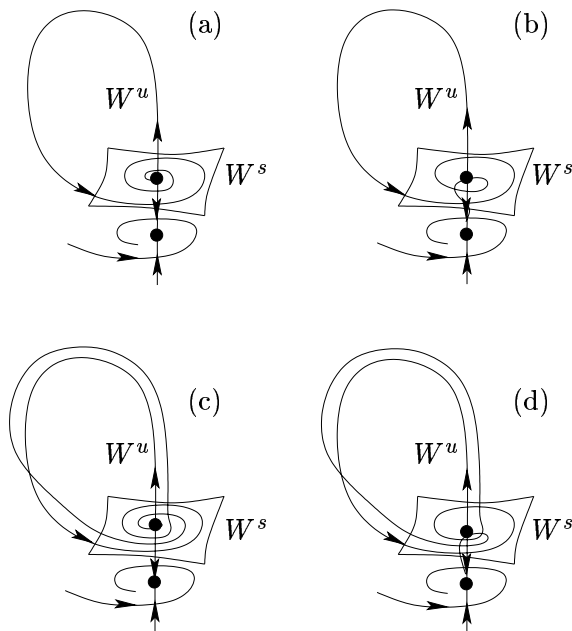


Figure 2. Excitability near a homoclinic orbit: a 1-homoclinic orbit (a) and nearby 1-pulse excitability (b); a 2-homoclinic orbit (c) and nearby 2-pulse excitability (d).

4. HOMOCLINIC TEETH

We now consider the structures in the two-dimensional (K, ω) -plane that give rise to multipulse excitability. To that end, we detected and continued different types of bifurcations, that is, qualitative changes of the system's dynamics. This lead to a picture of the *bifurcation diagram*, which consists of bifurcation curves dividing the (K, ω) -plane into regions of different dynamics of the laser; see Ref. [18] for more information on the bifurcation diagram of Eqs. (1).

Figure 3(a) shows what we call a ‘homoclinic tooth’, bounded by the saddle-node curve S and the homoclinic curve h^1 . There are two noncentral saddle-node homoclinic bifurcation points where S and h^1 meet. To the right of S and just above or below the homoclinic tooth the system is 1-pulse excitable.

The curve ns indicates where the saddle-quantity (the sum of the real eigenvalue and the real part of the complex pair of eigenvalues of the Jacobian at the saddle) is zero. The point where ns crosses h^1 is called a Belyakov bifurcation.¹⁴ To the left of the curve ns the homoclinic bifurcation along h^1 is a simple Shilnikov (saddle-focus homoclinic) bifurcation where a single attracting periodic orbit bifurcates. To the right of ns crossing h^1 corresponds to a chaotic Shil’nikov bifurcation with an infinite number of saddle periodic orbits of different periods close to the curve h^1 ; see, for example, Ref. [23].

These periodic orbits are created in n -homoclinic bifurcation curves h^n for any $n = 1, 2, 3, \dots$, which are organised in an intriguing way inside the homoclinic tooth. The enlargement in Fig. 3(b) shows a structure of

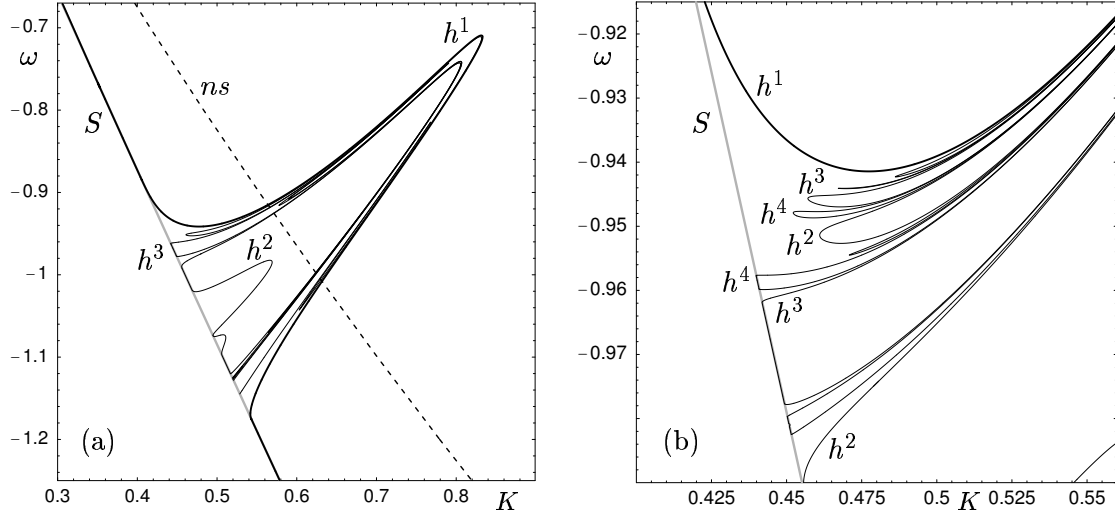


Figure 3. A homoclinic tooth (a) bounded by a curve h^1 of 1-homoclinic orbits is filled with regions, bounded by curves h^n of n -homoclinic orbits, where the laser is multipulse excitable, as is shown in the enlargement (b). Here $\alpha = 2.0$; compare Fig. 4.

smaller and smaller regions bounded by curves h^n . Many of these regions come close to the saddle-node bifurcation curve S and some are even attached to S at further noncentral saddle-node homoclinic bifurcation points. Inside a regions bounded by h^n the laser produces an n -pulse as the result of a single, suitable perturbation. We remark that these regions for $n \leq 3$ are so large that they could be experimentally resolved.²⁰

Since every intersection point of a homoclinic bifurcation curve h^n with ns is a new Belyakov point, the overall picture is that of an intricate arrangement of smaller and smaller regions. The details of this structure are still not known.¹⁵ For certain parameter values near h^1 above ns the excitable response is in fact chaotic: the trajectory may wander between a huge number of coexisting unstable orbits before settling back to the stable steady state.

We now discuss how the homoclinic teeth evolve with the linewidth enhancement factor α . This parameter describes how much the refractive index of the semiconductor material changes with the strength of the electric field. This change of refractive index leads to a phase self-modulation, an amplitude-frequency coupling also called chirping.

It is well-established^{18, 24} that α has a large influence on the dynamics of the laser, all the way from $\alpha = 0.0$, the case of a solid state or gas laser, to semiconductor lasers with α values presently in the range of 1 to 10. Generally speaking, the dynamics and the bifurcation diagram in the (K, ω) -plane become more complicated for increasing α .

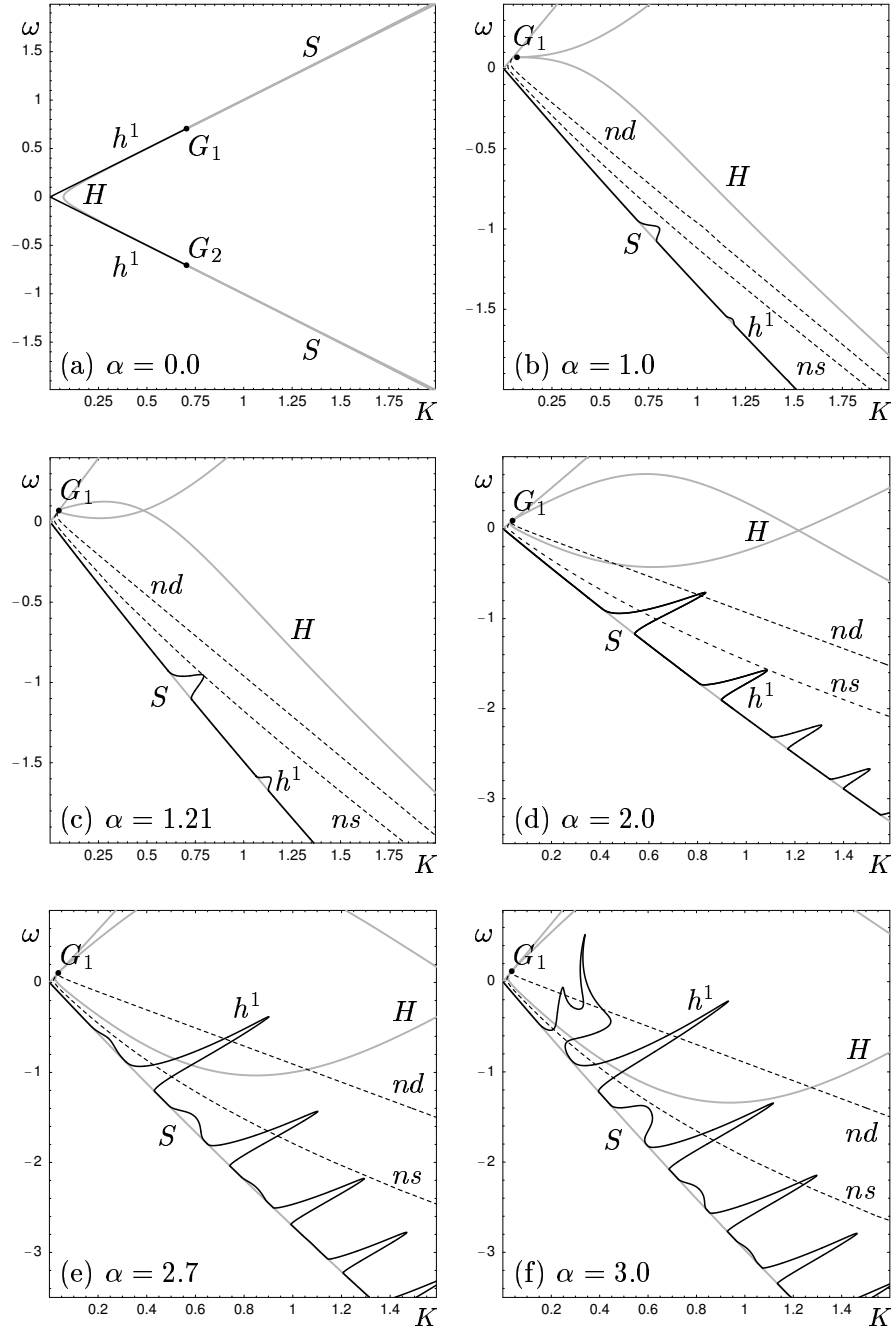


Figure 4. The homoclinic teeth grow with α and new teeth are born between old teeth, as is indicated in panels (a) to (f) for the indicated values of α .

This effect is also apparent when it comes to the dependence of homoclinic teeth on α , as is shown in Fig. 4. We plot saddle-node and Hopf bifurcation curves, denoted by S and H , which are known to meet at special points G_1 and G_2 ; see Ref. [18] for details. Supercritical parts of these curves (where attractors bifurcate) are plotted solid, while subcritical parts are plotted lighter. We also show the homoclinic bifurcation curve h^1 and the neutral saddle curve ns and a curve nd . At nd the real eigenvalue of the Jacobian at the saddle-focus is twice as large in modulus as the real part of the complex pair; this marks another transition in the type of Shilnikov bifurcation,²³ which we will not discuss here in detail.

For $\alpha = 0.0$ [Fig. 4(a)], that is, for solid state and gas lasers, there are no homoclinic teeth. (Note that the bifurcation diagram is symmetric in this case.) Small homoclinic teeth can be seen for $\alpha = 1.0$ [Fig. 4(b)], but they are all below the curve ns . Approximately at $\alpha = 1.21$ [Fig. 4(c)] the first and largest tooth touches ns , leading to the creation of Belyakov points for larger values of α . At $\alpha = 2.0$ [Fig. 4(d)] the second tooth just reached ns . The first and larger tooth is the one shown in more detail in Fig. 3. As the teeth keep growing, we see the appearance of new teeth between old ones, for example, for $\alpha = 2.7$ [Fig. 4(e)]. While all teeth keep growing, the small tooth closest to G_1 in Fig. 4(e) grows rapidly into a rather intricate shape for $\alpha = 3.0$ [Fig. 4(f)].

There appears to be a complicated interplay between the homoclinic teeth and the further curves of n -homoclinic bifurcations inside them. In particular, it is a surprising fact that multipulse excitability can also be found in homoclinic teeth in the absence of Belyakov bifurcations for $\alpha < 1.21$. The intricate structure of n -homoclinic bifurcations will be discussed in detail elsewhere.

5. CONCLUSIONS

We showed that multipulse excitability can be found inside the locking range of an optically injected semiconductor laser. More precisely, this phenomenon was found in what we call homoclinic teeth, in regions bounded by curves of n -homoclinic bifurcations. The role played by special points, namely noncentral saddle-node homoclinic bifurcations and Belyakov bifurcations, in the organization of these bifurcations was highlighted. We showed that the homoclinic teeth grow in size and number with the linewidth enhancement factor α .

Our results indicate the possibility of measuring multipulse excitability experimentally. The best strategy for this may be to use a laser with relatively large α and try to find a 2-pulse response to a single perturbation. We expect that if the laser is operated not too close to its locking boundary, given by the saddle-node curve S , spontaneous emission noise should not be able to trigger a pulse. A multipulse response could then be triggered by an externally applied perturbation, for example, in the form of an optical pulse.

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